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Masses of light tetraquarks and scalar mesons in the relativistic quark model

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Masses of the ground state light tetraquarks are dynamically calculated in the framework of the relativistic diquark-antidiquark picture. The internal structure of the diquark is taken into account by calculating the form factor of the diquark-gluon interaction in terms of the overlap integral of the diquark wave functions. It is found that scalar mesons with masses below 1 GeV: $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ agree well with the light tetraquark interpretation.

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The consistent theoretical understanding of the light meson sector remains an important problem already for many years. [1] An extensive analysis of the data on highly excited light non-strange meson states up to a mass of 2400 MeV collected by Crystal Barrel experiment at LEAR (CERN) has been published [1]. Classification of these new data requires better theoretical description of light meson mass spectra. This is especially important, since light exotic states (such as tetraquarks, glueballs, hybrids) within quantum chromodynamics (QCD) are expected to have masses in this range [2, 3, 4]. Particular interest is focused on scalar mesons, their properties and abundance. A generally accepted consistent picture has not yet emerged. Experimental and theoretical evidence [3] for the existence of $f_0(600)(\sigma)$, $K_0^*(800)(\kappa)$, $f_0(980)$ and $a_0(980)$ indicates that lightest scalars form a full $SU(3)$ flavour nonet. A peculiar feature of their mass spectrum is the inversion of the mass ordering, which cannot be naturally understood in the $q\bar{q}$ picture. This fact stimulated various alternative interpretations of light scalars as four quark states (tetraquarks) [5, 6] in particular diquark-antidiquark bound states [7]. The proximity of f_0/a_0 to the $K\bar{K}$ threshold led to the $K\bar{K}$ molecular picture [8].

In this paper we calculate the masses of the ground state ($\langle \mathbf{L}^2\rangle$=0) light tetraquarks as diquark-antidiquark bound states in the relativistic quark model based on the quasipotential approach in quantum chromodynamics. Following [5, 7] the diquark is taken in the colour antitriplet state. Recently, in the framework of the same model [9, 10] we investigated the mass spectra of heavy tetraquarks. It was found that many of the newly observed charmonium-like states [11] above open charm threshold, including explicitly exotic charged states, could be interpreted as tetraquark states with hidden charm. In the present analysis of light tetraquarks we use, as previously, the diquark-antidiquark picture to reduce

[1] A vast literature on the light meson spectroscopy is available. Therefore we mostly refer to the recent reviews where the references to earlier review and original papers can be found.

a complicated relativistic four-body problem to the subsequent two more simple two-body problems. The first step involves the calculation of the masses, wave functions and form factors of the diquarks, composed from light quarks. At the second step, a light tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure given by the form factor of the diquark-gluon interaction in terms of the diquark wave functions.

In the quasipotential approach and diquark-antidiquark picture of tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [12]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{E_1E_2}{E_1+E_2}=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are given by

$$E_1=\frac{M^2-m_2^2+m_1^2}{2M},\quad E_2=\frac{M^2-m_1^2+m_2^2}{2M}. \tag{3}$$

Here, $M=E_1+E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks ($q=u,d$ and s) which form the diquark or of the diquark (d) and antidiquark ($\bar{d}'$) which form the light tetraquark (T), and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. For the quark-quark interaction in a diquark we use the relation $V_{qq}=V_{q\bar{q}}/2$ arising under the assumption of an octet structure of the interaction from the difference in the qq and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark. The quasipotential is then defined as follows [9].

(a) For the quark-quark (qq, qs, ss) interactions, $V(\mathbf{p},\mathbf{q};M)$ reads

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{1}{2}\left[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu+V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\right].$$

Here, α_s is the QCD coupling constant; $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge,

$$D^{00}(\mathbf{k})=-\frac{4\pi}{\mathbf{k}^2},\quad D^{ij}(\mathbf{k})=-\frac{4\pi}{k^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right),\quad D^{0i}=D^{i0}=0, \tag{6}$$

and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors,

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{7}$$

with $\epsilon(p)=\sqrt{\mathbf{p}^2+m^2}$.

The effective long-range vector vertex of the quark is defined [13] by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu,\qquad \tilde{k}=(0,\mathbf{k}), \tag{8}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In configuration space the vector and scalar confining potentials in the nonrelativistic limit [12] reduce to

$$\begin{aligned}V^V_{\rm conf}(r)&=(1-\varepsilon)V_{\rm conf}(r),\\ V^S_{\rm conf}(r)&=\varepsilon V_{\rm conf}(r),\end{aligned} \tag{9}$$

with

$$V_{\rm conf}(r)=V^S_{\rm conf}(r)+V^V_{\rm conf}(r)=Ar+B, \tag{10}$$

where ε is the mixing coefficient.

(b) For the diquark-antidiquark ($d\bar{d}'$) interaction, $V(\mathbf{p},\mathbf{q};M)$ is given by

$$\begin{aligned}V(\mathbf{p},\mathbf{q};M)=&\frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_dE_d}}\frac{4}{3}\alpha_sD^{\mu\nu}(\mathbf{k})\frac{\langle\bar{d}'(P')|J_\nu|\bar{d}'(Q')\rangle}{2\sqrt{E_{d'}E_{d'}}}\\ &+\psi^*_d(P)\psi^*_{\bar{d}'}(P')\left[J_{d;\mu}J^\mu_{\bar{d}'}V^V_{\rm conf}(\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\right]\psi_d(Q)\psi_{\bar{d}'}(Q'),\end{aligned} \tag{11}$$

where $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the finite size of the diquark $\Big[P^{(')}=(E_{d^{(')}},\pm\mathbf{p})$ and $Q^{(')}=(E_{d^{(')}},\pm\mathbf{q})$, $E_d=(M^2-M^2_{d'}+M^2_d)/(2M)$ and $E_{d'}=(M^2-M^2_d+M^2_{d'})/(2M)\Big]$.

The diquark state in the confining part of the diquark-antidiquark quasipotential (11) is described by the wave functions

$$\psi_d(p)=\begin{cases}1 & \text{for a scalar diquark,}\\ \varepsilon_d(p) & \text{for an axial-vector diquark,}\end{cases} \tag{12}$$

where the four-vector

$$\varepsilon_d(p)=\left(\frac{(\boldsymbol{\varepsilon}_d\cdot\mathbf{p})}{M_d},\boldsymbol{\varepsilon}_d+\frac{(\boldsymbol{\varepsilon}_d\cdot\mathbf{p})\mathbf{p}}{M_d(E_d(p)+M_d)}\right),\qquad \varepsilon^\mu_d(p)p_\mu=0, \tag{13}$$

TABLE I: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[\ldots]$ and symmetric $\{\ldots\}$ in flavour, respectively.

Quark content	Diquark type	Mass [14] our	[15] NJL	[16] BSE	[17] BSE	[18] Lattice
$[u,d]$	S	710	705	737	820	694(22)
$\{u,d\}$	A	909	875	949	1020	806(50)
$[u,s]$	S	948	895	882	1100	
$\{u,s\}$	A	1069	1050	1050	1300	
$\{s,s\}$	A	1203	1215	1130	1440	

is the polarization vector of the axial-vector diquark with momentum $\mathbf{p}$, $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$, and $\varepsilon_d(0) = (0, \boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for a scalar diquark,} \\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu \tilde{k}_\nu & \text{for an axial-vector diquark.} \end{cases} \tag{14}$$

Here, the antisymmetric tensor Σ_μ^ν is defined by

$$(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu), \tag{15}$$

and the axial-vector diquark spin $\mathbf{S}_d$ is given by $(S_{d;k})_{il} = -i\varepsilon_{kil}$; μ_d is the total chromomagnetic moment of the axial-vector diquark.

The constituent quark masses $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV, fixed previously [13], have values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [12] and the heavy-quark expansion. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J - states [12]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

At the first step, we take the masses and form factors of the light diquarks from the previous consideration of light diquarks in heavy baryons [14]. The form factor $F(r)$ entering the vertex of the diquark-gluon interaction was expressed through the overlap integral of the diquark wave functions. Our estimates showed that this form factor can be approximated with high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}. \tag{16}$$

The values of the masses and parameters ξ and ζ for light scalar diquark $[\cdots]$ and axial vector diquark $\{\cdots\}$ ground states are given in Table I, II.

TABLE II: Parameters ξ and ζ for ground state light diquarks.

Quark content	Diquark type	ξ (GeV)	ζ (GeV^2)
$[u,d]$	S	1.09	0.185
$\{u,d\}$	A	1.185	0.365
$[u,s]$	S	1.23	0.225
$\{u,s\}$	A	1.15	0.325
$\{s,s\}$	A	1.13	0.280

At the second step, we calculate the masses of light tetraquarks considered as the bound states of a light diquark and antidiquark. For the potential of the diquark-antidiquark interaction (11) we get in configuration space [10]

$$\begin{aligned}
V(r) \;=\; & \hat{V}_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{2}\Biggl\{\left[\frac{1}{E_1(E_1+M_1)}+\frac{1}{E_2(E_2+M_2)}\right]\frac{\hat{V}'_{\rm Coul}(r)}{r}-\left[\frac{1}{M_1(E_1+M_1)}\right.\\
&\left.+\frac{1}{M_2(E_2+M_2)}\right]\frac{V'_{\rm conf}(r)}{r}+\frac{\mu_d}{2}\left(\frac{1}{M_1^2}+\frac{1}{M_2^2}\right)\frac{V'^V_{\rm conf}(r)}{r}\Biggr\}\mathbf{L}\cdot(\mathbf{S}_1+\mathbf{S}_2)\\
&+\frac{1}{2}\Biggl\{\left[\frac{1}{E_1(E_1+M_1)}-\frac{1}{E_2(E_2+M_2)}\right]\frac{\hat{V}'_{\rm Coul}(r)}{r}-\left[\frac{1}{M_1(E_1+M_1)}-\frac{1}{M_2(E_2+M_2)}\right]\\
&\times\frac{V'_{\rm conf}(r)}{r}+\frac{\mu_d}{2}\left(\frac{1}{M_1^2}-\frac{1}{M_2^2}\right)\frac{V'^V_{\rm conf}(r)}{r}\Biggr\}\mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2)\\
&+\frac{1}{E_1E_2}\Biggl\{\mathbf{p}\left[\hat{V}_{\rm Coul}(r)+V^V_{\rm conf}(r)\right]\mathbf{p}-\frac{1}{4}\Delta V^V_{\rm conf}(r)+\hat{V}'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r}\\
&+\frac{1}{r}\left[\hat{V}'_{\rm Coul}(r)+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)V'^V_{\rm conf}(r)\right]\mathbf{L}\cdot(\mathbf{S}_1+\mathbf{S}_2)\\
&+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}-\frac{E_2}{M_2}\right)\frac{V'^V_{\rm conf}(r)}{r}\mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2)\\
&+\frac{1}{3}\left[\frac{1}{r}\hat{V}'_{\rm Coul}(r)-\hat{V}''_{\rm Coul}(r)+\frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^V_{\rm conf}(r)-V''^V_{\rm conf}(r)\right)\right]\\
&\times\left[\frac{3}{r^2}(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})-\mathbf{S}_1\cdot\mathbf{S}_2\right]\\
&+\frac{2}{3}\left[\Delta\hat{V}_{\rm Coul}(r)+\frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_1\cdot\mathbf{S}_2\Biggr\}, \qquad (17)
\end{aligned}$$

where

$$\hat{V}_{\rm Coul}(r)=-\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. Here, $\mathbf{S}_{1,2}$ and

L are the spin operators of diquark and antidiquark and the operator of the relative orbital angular momentum. In the following we choose the total chromomagnetic moment of the axial-vector diquark $\mu_d = 0$. Such a choice appears to be natural, since the long-range chromomagnetic interaction of diquarks proportional to μ_d then also vanishes in accordance with the flux-tube model.

We substitute the diquark-antidiquark interaction potential (17) in the wave equation (1) and solve it numerically in configuration space. The calculated masses of the ground states of light tetraquarks considered as light diquark-antidiquark bound systems are given in Tables III and IV. In Table III we present masses for light unflavoured tetraquarks (without or with hidden strangeness) and in Table IV - masses of strange tetraquarks. Possible experimental candidates for such states are also given.

In the diquark-antidiquark picture of tetraquarks both scalar S (antisymmetric in flavour $[\ldots]$) and axial vector A (symmetric in flavour $\{\ldots\}$) diquarks are considered. Therefore we get the following structure of the light tetraquark ground ($1S$) states (C is defined only for neutral self-conjugated states):

- Two states with $J^{PC} = 0^{++}$:

$$X(0^{++}) = S\bar{S}$$
$$X(0^{++\prime}) = A\bar{A}$$

- Three states with $J^{PC} = 1^{+\pm}$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}(S\bar{A} + \bar{S}A)$$
$$X(1^{+-}) = \frac{1}{\sqrt{2}}(S\bar{A} - \bar{S}A)$$
$$X(1^{+-\prime}) = A\bar{A}$$

- One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = A\bar{A}.$$

The lightest $S\bar{S}$ scalar (0^{++}) tetraquark states form the SU(3) flavour nonet: one tetraquark ($[ud][\bar{u}\bar{d}]$) with neither open or hidden strangeness (electric charge $Q = 0$ and isospin $I = 0$); four tetraquarks ($[sq][\bar{u}\bar{d}]$, $[\bar{s}\bar{q}][ud]$, $q = u, d$) with open strangeness ($Q = 0, \pm 1$, $I = \frac{1}{2}$) and four tetraquarks ($[sq][\bar{s}\bar{q}']$) with hidden strangeness ($Q = 0, \pm 1$, $I = 0, 1$). Since we neglect in our model the mass difference of u and d quarks and electromagnetic interactions, the corresponding tetraquarks will be degenerate in mass.

From Tables III and IV we see that the diquark-antidiquark picture can provide a natural explanation for the inversion of masses of light scalar 0^+ mesons. Indeed all lightest experimentally observed scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be interpreted in our model as light tetraquarks composed from a scalar diquark and antidiquark ($S\bar{S}$). Therefore, the $f_0(980)$ and $a_0(980)$ tetraquarks contain, in comparison to the $q\bar{q}$ picture, an additional pair of strange quarks which gives a natural explanation why their masses are heavier than the strange $K_0^*(800)$ (κ). Note that physical neutral scalar states with $I = 0$ are in fact mixtures of pure tetraquark f_0 and σ_0 states

$$|f\rangle \ = \ \cos\varphi|f_0\rangle + \sin\varphi|\sigma_0\rangle,$$

TABLE III: Masses of light unflavored diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Theory mass	Experiment [19] $I=0$	mass	$I=1$	mass
$(qq)(\bar q\bar q)$						
0^{++}	$S\bar S$	596	$f_0(600)$ (σ)	400-1200		-
$1^{+\pm}$	$(S\bar A \pm \bar S A)/\sqrt{2}$	672				
0^{++}	$A\bar A$	1179	$f_0(1370)$	1200-1500		
1^{+-}	$A\bar A$	1773				
2^{++}	$A\bar A$	1915	$f_2(1910)$	1903(9)		
			$f_2(1950)$	1944(12)		
$(qs)(\bar q\bar s)$						
0^{++}	$S\bar S$	992	$f_0(980)$	980(10)	$a_0(980)$	984.7(12)
1^{++}	$(S\bar A + \bar S A)/\sqrt{2}$	1201	$f_1(1285)$	1281.8(6)	$a_1(1260)$	1230(40)
1^{+-}	$(S\bar A - \bar S A)/\sqrt{2}$	1201	$h_1(1170)$	1170(20)	$b_1(1235)$	1229.5(32)
0^{++}	$A\bar A$	1480	$f_0(1500)$	1505(6)	$a_0(1450)$	1474(19)
1^{+-}	$A\bar A$	1942	$h_1(1965)$	1965(45)	$b_1(1960)$	1960(35)
2^{++}	$A\bar A$	2097	$f_2(2010)$	2011(70)	$a_2(1990)$	2050(45)
			$f_2(2140)$	2141(12)	$a_2(2080)$	2100(20)
$(ss)(\bar s\bar s)$						
0^{++}	$A\bar A$	2203	$f_0(2200)$	2189(13)		-
1^{+-}	$A\bar A$	2267	$h_1(2215)$	2215(40)		-
2^{++}	$A\bar A$	2357	$f_2(2340)$	2339(60)		-

TABLE IV: Masses of strange diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^P	Diquark content	Theory mass	Experiment [19] $I=\frac{1}{2}$	mass
$(qq)(\bar s\bar q)$ or $(sq)(\bar q\bar q)$				
0^+	$S\bar S$	730	$K_0^*(800)$ (κ)	672(40)
1^+	$(S\bar A \pm \bar S A)/\sqrt{2}$	1057		
0^+	$A\bar A$	1332	$K_0^*(1430)$	1425(50)
1^+	$A\bar A$	1855		
2^+	$A\bar A$	2001	$K_2^*(1980)$	1973(26)

$$|\sigma\rangle \;=\; -\sin\varphi|f_0\rangle + \cos\varphi|\sigma_0\rangle, \tag{18}$$

where

$$f_0 = \frac{1}{\sqrt{2}}([su][\bar s\bar u] + [sd][\bar s\bar d]); \qquad \sigma_0 = [ud][\bar u\bar d].$$

However the $f-\sigma$ mixing is small because the Zweig rule is expected to hold in the physical

mass spectrum [7]. Our results support such conclusion, since we get good agreement with experimental data already without such a mixing.

The other scalar tetraquark states can be composed from an axial vector diquark and antidiquark ($A\bar{A}$). Their masses are predicted to be approximately 600 MeV heavier than the $S\bar{S}$ tetraquarks. The diquark-antidiquark composition also naturally explains the experimentally observed proximity of masses of the unflavored $a_0(1450)$, $f_0(1500)$ and strange $K_0^*(1430)$ scalars. Let us note that quark-antiquark scalar states are predicted in our model to have masses around 1200 MeV ($q\bar{q}$) and 1400 MeV ($q\bar{s}$).

The axial vector 1^+ states can be composed both from scalar and axial vector diquark and antidiquark ($(S\bar{A} \pm \bar{S}A)/\sqrt{2}$) and from an axial vector diquark and antidiquark ($A\bar{A}$), respectively. Our model predicts rather low mass values of the former states composed from light quarks ($\{ud\}[\bar{u}\bar{d}] \pm \{\bar{u}\bar{d}\}[ud])/\sqrt{2}$, 672 MeV, and of their strange partner ($[qs]\{\bar{u}\bar{d}\} \pm [\bar{q}\bar{s}]\{ud\}$), 1057 MeV. Such axial vector states are not observed experimentally. Note that the recent study [20] also indicates that corresponding light tetraquarks should have masses below 1 GeV. On the other hand, there are several candidates for the axial vector ($\{qs\}[\bar{q}\bar{s}] \pm \{\bar{q}\bar{s}\}[us])/\sqrt{2}$ tetraquarks both in isospin $I = 1$ ($a_1(1260)$, $b_1(1235)$) and $I = 0$ ($f_1(1285)$, $h_1(1170)$) channels. However, ordinary $q\bar{q}$ axial vector mesons are expected to have close masses. Therefore the observed states can in principle be mixtures of $q\bar{q}$ and tetraquark states. There are also possible experimental candidates for the axial vector 1^{+-} $\{qs\}\{\bar{q}\bar{s}\}$ tetraquark with isospin $I = 1$, $b_1(1960)$, and with $I = 0$, $h_1(1965)$, as well as for the 1^{+-} $\{ss\}\{\bar{s}\bar{s}\}$ tetraquark, $h_1(2215)$.

The diquark-antidiquark ground state tetraquark, composed from an axial vector diquark and antidiquark ($A\bar{A}$), can also be in the tensor 2^+ state. The possible experimental candidates are the following: a $\{qq\}\{\bar{q}\bar{q}\}$ tetraquark with $I = 0$, $f_2(1910)$ or $f_2(1950)$; the $\{qs\}\{\bar{q}\bar{s}\}$ tetraquark with $I = 1$, $a_2(1990)$ or $a_2(2080)$, and with $I = 0$, $f_2(2010)$ or $f_2(2140)$; for the $\{ss\}\{\bar{s}\bar{s}\}$ tetraquark $f_2(2340)$; for the $\{qq\}\{\bar{q}\bar{s}\}$ tetraquark $K_2^*(1980)$.

There remains the important problem of describing simultaneously the mass spectrum and decay rates of the light and heavy nonets of scalar mesons within the relativistic quark model. This requires the inclusion of instanton-induced mixing terms [7] and will be investigated in future.

In summary, we calculated the masses of the ground state light tetraquarks in the diquark-antidiquark picture. In distinction with previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equations. The diquark structure was taken into account by using diquark-gluon form factors in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of hadron properties. It was found that the lightest scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be naturally described in our model as diquark-antidiquark bound systems.

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